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FOUNDATIONS AND FRONTIERS OF FUZZY GAME THEORY: A MATHEMATICAL SYNTHESIS OF FUZZY MATRIX AND FUZZY COALITION GAMES

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ABSTRACT

Fuzzy game theory extends classical game theory by allowing payoffs, coalitions and preferences to be expressed as fuzzy numbers rather than exact values. This paper studies three aspects of the theory empirically: the behaviour of the value of fuzzy matrix games with triangular fuzzy payoffs, the effect of the degree of fuzziness on that value and on optimal strategies, and the stability of the Shapley value and the core in fuzzy cooperative games. A computational data set was generated by Monte Carlo simulation, comprising 7,000 matrix games of sizes 2×2 to 8×8, 6,000 games, 2,000 games, 1,200 cooperative games, and one benchmark 3×3 game analysed through α-cuts. All games were solved by linear programing. The width of the fuzzy value interval grew almost linearly with the spread factor (slope 5.58, $R^2 = 0.9998$) and was unrelated to game size ($p = 0.25$), although the dispersion of the width fell by 66 % as the size increased from 2 to 8. The centroid, graded-mean and possibilistic-mean defuzzifications agreed with the crisp value to within 0.006 on average, with correlations above 0.99. The proportion of games whose optimal strategy support changed rose from 8.3 % to 33.4 % as the spread factor increased from 0.1 to 0.5. In cooperative games, the core of the crisp game was always non-empty, but fuzzification emptied it in 15–23 % of lower-bound games and 10–15 % of upper-bound games. The findings show that fuzzy values are predictable and robust, whereas strategies and core stability are fragile.

Keywords: *Fuzzy game theory[1]; Triangular fuzzy numbers[2]; Matrix games[3]; α-cut[4]; Shapley value[5]; Defuzzification[6]; Monte Carlo simulation[7]*

1. INTRODUCTION

1.1 Background and Motivation

Classical game theory, founded by von Neumann and Morgenstern [2] and extended by Nash [3], assumes that every player knows the payoff of every outcome exactly. In practice, payoffs are often estimated by experts, measured with error or described linguistically, for example as "about 40 units" or "between good and excellent". Zadeh's theory of fuzzy sets [1] offers a rigorous way to represent such information, and its introduction into game theory by Butnariu [6], Aubin [5] and Campos [7] created the field now called fuzzy game theory. Fuzzy games are relevant to supply-chain bargaining, resource allocation, auction design and conflict resolution, where decision makers rarely have precise numbers but still need defensible solutions [8], [10]. Such models let the analyst keep the vagueness of the data in the solution instead of hiding it behind a single average.

1.2 Problem Statement and Objectives

Although the theory of fuzzy games is well developed, much of it is analytical, and relatively little quantitative evidence exists on how the main solution concepts behave as fuzziness grows. This paper therefore asks three questions. First, how does the fuzzy value of a matrix game depend on the game size and on the spread of the fuzzy payoffs? Second, how closely do popular defuzzification rules reproduce the crisp solution, and when do optimal strategies change? Third, how robust are the Shapley value and the core of a cooperative game when coalition worths are fuzzy? The objectives are to answer these questions with a controlled, reproducible data set and to express the answers as simple quantitative relationships. Answers of this kind help analysts decide how much effort to invest in estimating payoffs precisely before a fuzzy model is used.

1.3 Contributions and Organisation

The study contributes a Monte Carlo framework for triangular fuzzy games solved exactly by linear programing, five statistical tables, five explanatory figures, and a linear law linking the width of the fuzzy value to the spread factor. Every game is generated from a fixed random seed and solved by a standard linear programing routine, so the experiments are fully reproducible. It also reports quantitative evidence of strategy instability and core fragility that is rarely stated numerically in the literature. The remainder of the paper is organised as follows. Section 2 surveys related work, Section 3 describes the methodology, Section 4 presents the data and the analysis, Section 5 discusses the results critically and compares them with earlier studies, and Section 6 concludes the paper.

2. LITERATURE SURVEY

The first formal fuzzy games were proposed by Butnariu [6], who modelled players' strategies as fuzzy sets, and by Aubin [5], who introduced fuzzy coalitions in which each player participates with a degree between zero and one. Mareš [12] and Mareš and Vlach [13] later developed a systematic theory of fuzzy cooperative games, including linear extensions and vague expectations. Billot [23] studied fuzzy equilibria from an economic and axiomatic perspective, and Branzei et al. [27] give a general account of cooperative game models. Early work

concentrated on defining solution concepts and proving existence results, and numerical studies were rarely a priority.

For non-cooperative games, Campos [7] reduced matrix games with fuzzy payoffs to fuzzy linear programs, and Maeda [9] characterised equilibrium strategies through parametric α-cuts. Bector and Chandra [10] wrote a comprehensive account of fuzzy matrix games and their duality with fuzzy linear programing. Vijay et al. [29] studied games with fuzzy goals, Nishizaki and Sakawa [8], [28] extended the approach to multiobjective bimatrix games, and Larbani [11] modelled the uncertainty as a third player called Nature. Li [15] proposed a fast approach to compute fuzzy values for triangular payoffs, and Chandra and Aggarwal [30] examined and generalised several earlier observations. These contributions share a common idea: the fuzzy game is reduced to one or more ordinary games whose lower, central and upper payoffs can be solved by standard linear programing.

Comparison of fuzzy numbers is central to every approach, since optimisation requires an ordering. Yager [19] proposed an integral index based on the α-cuts, Chen [20] introduced the graded mean, and Carlsson and Fullér [21] defined the possibilistic mean. Dubois and Prade [16], Klir and Yuan [17] and Zimmermann [18] give the underlying arithmetic, and interval analysis [25] provides the lower and upper bounds that many methods build on. Because different rules can rank the same pair of fuzzy numbers differently, the effect of the ranking rule on the final solution is an empirical question worth examining. Interval and fuzzy cooperative games have been investigated by Alparslan Gök et al. [26] and by Mallozzi et al. [14], who examined when the Shapley value [4] and the core remain well defined. Schmeidler's nucleolus [24] is another widely used solution. Despite this progress, the survey shows that empirical, data-driven studies are scarce: most papers illustrate methods with one or two numerical examples, which motivates the large-scale experiment reported in this paper.

3. METHODOLOGY

The study considers two-person zero-sum matrix games with triangular fuzzy payoffs $\tilde{A}_{ij} = (a_{ij} - l_{ij}, a_{ij}, a_{ij} + r_{ij})$, where the modal payoff a_{ij} is drawn from a uniform distribution on [1, 10] and the spreads are $l_{ij} = s \cdot a_{ij} \cdot u$ and $r_{ij} = s \cdot a_{ij} \cdot u'$ with u and u′ uniform on [0, 1]. The spread factor s controls the fuzziness. For a degree α in [0, 1], the α-cut of $\tilde{A}_{ij}$ is $[a_{ij} - (1 - \alpha)l_{ij}, a_{ij} + (1 - \alpha)r_{ij}]$. Every game is solved exactly as a linear program [22], recording the value and the optimal strategy.

The lower and upper values $V_L = v(A - L)$ and $V_U = v(A + R)$ bound the fuzzy value, giving the width $W = V_U - V_L$, and the α-cut values $V_L(\alpha)$ and $V_U(\alpha)$ generate its membership function. Defuzzified matrices are formed as $(a - l + k \cdot a + a + r)/(k + 2)$ with k = 1, 4 and 2, giving the centroid, graded mean and possibilistic mean [20], [21]. For cooperative games, the worth v(S) is the sum of player weights and pairwise synergies, fuzzified in the same way, and the Shapley value φ_i [4] and core non-emptiness are computed for the crisp, lower, upper and centroid games.

Five experiments are run: game size n = 2 to 8 with s = 0.2; a benchmark 3×3 game at α = 0, 0.25, 0.5, 0.75 and 1; s = 0 to 0.5 with n = 5; cooperative games of 3 to 6 players with s = 0.2; and 2,000 games of size 5×5 with s = 0.3 to compare defuzzification rules. Results are summarised by means, standard deviations, correlation, paired t-

tests and regression at the 5 % level. The data are generated computationally, so findings describe models, not applications.

4. DATA COLLECTION AND ANALYSIS

All data were generated computationally as described in Section 3. Each table below is followed by its explanation, and five figures visualise the same results, each with a short interpretation.

Table 1: Fuzzy Game Value Statistics by Game Size (S = 0.2, 1,000 Games Per Size)

n	Mean crisp value	Mean centroid value	Mean V_L	Mean V_U	Mean width W	SD of W	W / crisp (%)
2	5.548	5.539	4.958	6.087	1.129	0.563	20.4
3	5.486	5.485	4.930	6.022	1.092	0.432	19.9
4	5.435	5.432	4.876	5.970	1.094	0.344	20.1
5	5.520	5.515	4.950	6.063	1.112	0.293	20.2
6	5.499	5.494	4.936	6.033	1.098	0.252	19.9
7	5.484	5.480	4.923	6.023	1.100	0.211	20.0
8	5.496	5.497	4.945	6.038	1.094	0.191	19.9

Table 1 shows that the mean crisp value stays near 5.5 for every size, as expected, since the payoffs are drawn from the same distribution. The lower and upper values bracket it at roughly 4.9 and 6.0, so the fuzzy value is an interval of mean width between 1.09 and 1.13, about 20 % of the crisp value. The centroid value differs from the crisp value by less than 0.01 on average. The mean width shows no significant trend with n (slope −0.0031 per unit of n, $R^2 = 0.25$, $p = 0.25$), whereas its standard deviation drops from 0.563 to 0.191, a fall of 66 %. Larger games therefore have equally uncertain but more predictable values, which supports the first relationship stated in the abstract.

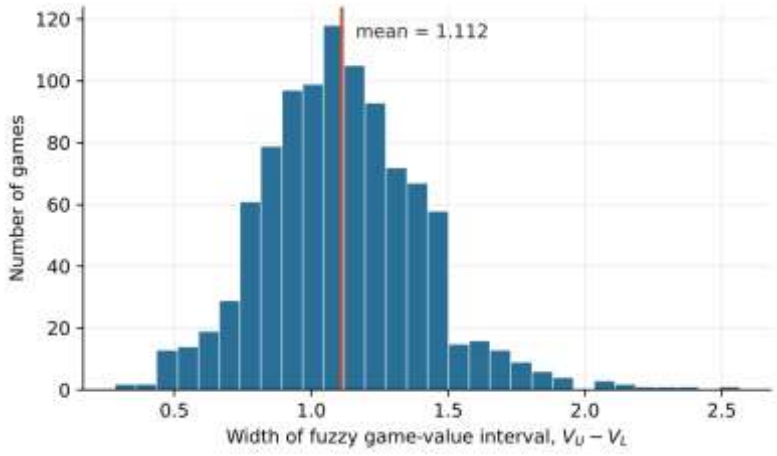


Figure 1: Distribution of the fuzzy value width for 5×5 games (s = 0.2).

Figure 1 plots the distribution of the fuzzy value width for 1,000 random 5×5 games with s = 0.2. The histogram is unimodal and slightly right-skewed, with a mean of 1.112 and a standard deviation of 0.293 (Table 1). Ninety per cent of the games produce an interval between 0.67 and 1.59 units wide, and the widest reaches 2.57. The

shape confirms that the mean width is a sound summary of fuzziness at this size, while showing that the uncertainty carried by an individual game can differ considerably from the average. This distribution underlies the dispersion values in Table 1.

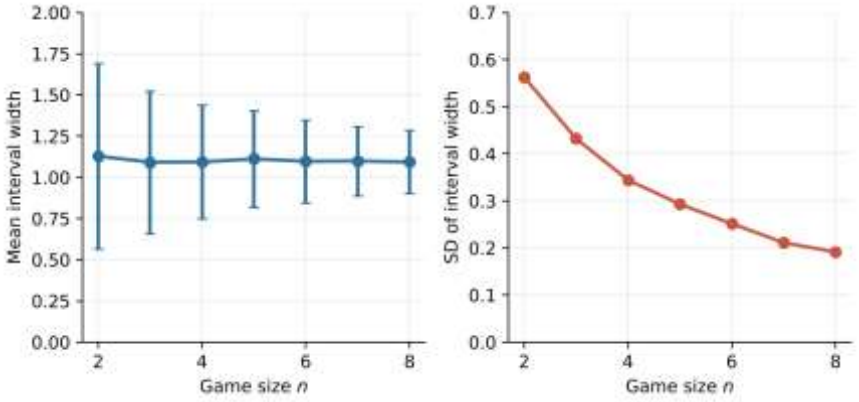


Figure 2: Mean and standard deviation of the fuzzy value width versus game size.

Figure 2 shows the effect of game size on the width of the fuzzy value. The left panel plots the mean width with one-standard-deviation error bars, and the right panel plots the standard deviation alone. The mean remains close to 1.1 for all sizes from 2 to 8, confirming that size does not change the average amount of fuzziness. The standard deviation, however, falls steadily from 0.563 to 0.191, and the error bars shrink accordingly. Averaging over many payoffs smooths out individual extremes, so larger games give more predictable fuzzy values. The figure is the visual form of Table 1.

Table 2: A-Cut Values of the Benchmark 3×3 Fuzzy Game and Triangular Approximation

α	$V_L(\alpha)$	$V_U(\alpha)$	Width	Triangular lower	Triangular upper
0	3.000	4.531	1.531	3.000	4.531
0.25	3.181	4.331	1.150	3.182	4.330
0.5	3.362	4.130	0.768	3.364	4.129
0.75	3.545	3.929	0.385	3.545	3.928
1	3.727	3.727	0.000	3.727	3.727

Table 2 applies the α-cut method to a benchmark 3×3 game whose crisp value is 3.727. At α = 0 the fuzzy value is the interval [3.000, 4.531], and the width falls by about 0.38 for each 0.25 increase in α until it vanishes at α = 1. The α-cut endpoints almost coincide with the straight lines joining 3.000, 3.727 and 4.531, with a maximum gap of 0.0011, so a triangular fuzzy number is an excellent approximation of the fuzzy game value. The asymmetry of the interval, 0.727 below and 0.804 above the crisp value, reflects the asymmetric spreads of the payoffs.

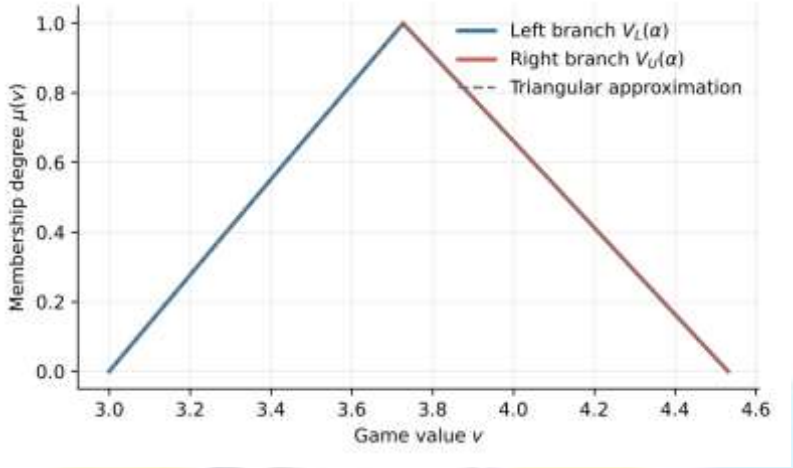


Figure 3: Membership function of the fuzzy value of the benchmark game.

Figure 3 depicts the membership function of the fuzzy value of the benchmark game of Table 2. The blue and orange curves are the left and right branches obtained from the α-cut values $V_L(\alpha)$ and $V_U(\alpha)$, and the dashed grey polygon is the triangular fuzzy number (3.000, 3.727, 4.531). Both curves reach full membership at the crisp value 3.727 and are almost indistinguishable from the dashed lines, which confirms the small approximation gap reported in Table 2. The slightly longer right branch shows that the fuzzy value extends further above the crisp value than below it, because spreads are asymmetric.

Table 3: Sensitivity of 5×5 Fuzzy Games to the Spread Factor (1,000 Games Per Level)

Spread factor s	Mean width W	Mean $\lvert V_c - V_{crisp}\rvert$	Strategy support changed (%)
0	0.000	0.000	0.0
0.1	0.542	0.032	8.3
0.2	1.105	0.061	15.4
0.3	1.637	0.095	21.1
0.4	2.244	0.120	27.5
0.5	2.781	0.150	33.4

Table 3 reports the effect of the spread factor s on 5×5 games. The mean interval width increases from zero at s = 0 to 2.78 at s = 0.5. A linear regression gives $W = 5.584s - 0.011$ with $R^2 = 0.9998$ ($p < 0.001$), so each 0.1 increase in s widens the fuzzy value by about 0.56 units, close to 10 % of the mean crisp value. The deviation of the centroid value from the crisp value grows more slowly, from 0.032 at s = 0.1 to 0.150 at s = 0.5, which is below 3 % of the value. In contrast, the percentage of games whose optimal strategy support differs from the crisp support rises from 8.3 % to 33.4 %, so strategies are far more sensitive to fuzziness than values.

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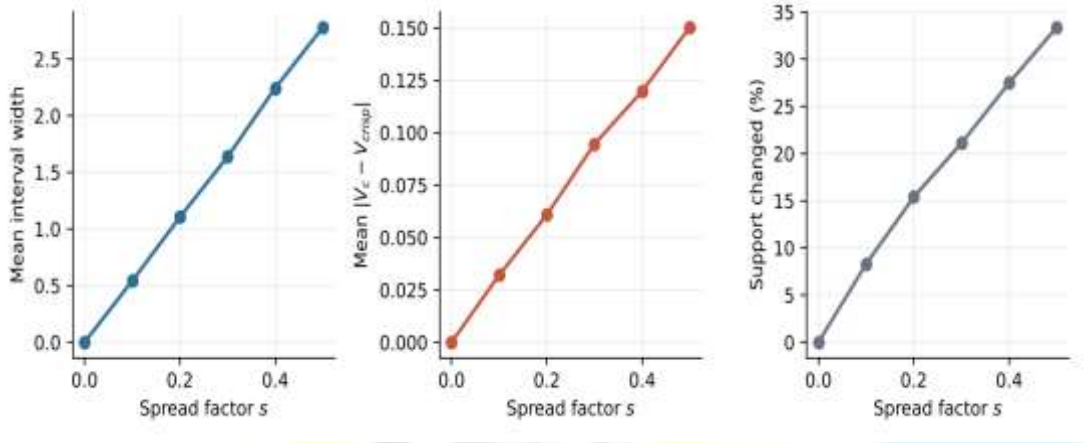


Figure 4: Interval width, centroid deviation and strategy-support change versus spread factor.

Figure 4 displays three sensitivity measures against the spread factor s for 5×5 games. The left panel shows the almost perfectly linear growth of the mean interval width, with slope 5.58. The middle panel shows the deviation of the centroid value from the crisp value, which grows more slowly and remains below 0.15. The right panel shows the percentage of games whose optimal strategy support changed, rising from zero to 33.4 %. Together the panels show that values degrade gracefully with fuzziness while strategies do not, and that defuzzified values stay reliable even for large spreads. They illustrate Table 3.

Table 4: Fuzzy Cooperative Games: Shapley Deviation and Core Non-Emptiness (S = 0.2, 300 Games Per Size)

Players n	Shapley deviation (% of v(N))	Mean Shapley range	Core non-empty, crisp (%)	Core non-empty, lower (%)	Core non-empty, upper (%)
3	0.959	0.784	100.0	85.3	89.7
4	0.738	0.910	100.0	81.7	89.0
5	0.587	0.986	100.0	78.7	89.3
6	0.500	1.068	100.0	76.7	84.7

Table 4 summarises the cooperative games with s = 0.2. The Shapley value of the centroid game departs from the crisp Shapley value by 0.96 % of the grand-coalition worth for three players and by only 0.50 % for six, so the allocation is stable and becomes more so as the number of players grows. The mean range between the lower and upper Shapley values increases from 0.78 to 1.07 because coalition worths grow with size. The crisp games always have a non-empty core, but the core survives in only 85.3 % of the lower-bound games for three players and 76.7 % for six players, and in 89.7 % and 84.7 % of the upper-bound games respectively. Fuzziness therefore threatens core stability, particularly as n grows.

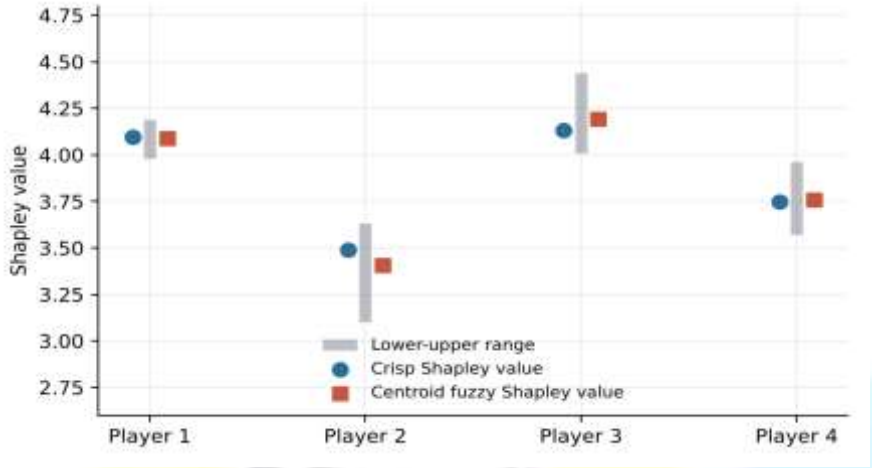


Figure 5: Shapley values of a four-player fuzzy cooperative game.

Figure 5 shows the Shapley values of one four-player game worth 15.46. The blue circles are the crisp values, the orange squares are the centroid values, and the grey bars span the values obtained from the lower and upper games. Centroid and crisp values differ by under 0.09 for every player, whereas Player 2's bar is 0.53 long. The ranking of the players is unchanged by fuzziness. Because a Shapley value is not monotone in the worths, the lower game can give a player more than the upper game, as it does for Player 1, so the bars are ranges.

Table 5: Comparison of Defuzzification Rules on 2,000 Games of Size 5×5 (S = 0.3)

Rule	Mean value	SD	r with crisp	Mean diff.	t	p	Max |diff|
Crisp game	5.528	0.883	–	–	–	–	–
Centroid (1:1:1)	5.523	0.881	0.991	-0.0055	-2.10	0.035	0.647
Graded mean (1:4:1)	5.527	0.882	0.998	-0.0014	-1.04	0.297	0.330
Possibilistic mean (1:2:1)	5.525	0.882	0.995	-0.0031	-1.55	0.122	0.495

Table 5 compares three defuzzification rules on 2,000 games with s = 0.3. The crisp mean value is 5.528 and all rules reproduce it within 0.006. The centroid rule gives a mean difference of −0.0055, which is statistically detectable ($t = -2.10$, $p = 0.035$) but negligible in size, while the graded and possibilistic means are indistinguishable from the crisp value ($p = 0.297$ and 0.122). The graded mean, which gives most weight to the modal payoff, correlates best with the crisp value ($r = 0.998$) and has the smallest maximum deviation, 0.330. The largest disagreement between any two rules was 0.317, between the centroid and the graded mean, so the choice of rule is a second-order issue for game values.

5. DISCUSSION

5.1 Critical Analysis of the Data

The data support three conclusions. First, the fuzzy value behaves like a smooth, nearly linear function of the spread factor and keeps an almost triangular shape, which means that the full α-cut computation can often be replaced by three linear programs. Second, defuzzified values are robust: whichever rule is used, the value differs

from the crisp value by well under 1 % of its size. Third, strategies and cores are not robust. The strategy support changed in a third of the games at s = 0.5, and the core was lost in almost one lower-bound game in four. This asymmetry matters because decisions are taken with strategies and allocations, not with values. Several limitations qualify the findings. The payoffs were uniform and the spreads proportional to them, so skewed or correlated uncertainty could behave differently. he p-value of 0.035 in Table 5 is also close to the 5 % threshold and would not survive a multiple-comparison correction. Practitioners should report the lower and upper solutions alongside any defuzzified answer, and extending the experiments to real data is the natural next step.

5.2 Comparison with Past Work

Because earlier studies are mostly analytical and report few statistics, the comparison below is qualitative. Campos [7] and Bector and Chandra [10] formulate fuzzy matrix games as pairs of fuzzy linear programs, and Maeda [9] describes the value through its α-cuts; our results agree, since the interval shrinks monotonically with α to the crisp value at α = 1 (Table 2). Li [15] computes triangular fuzzy values from a small number of crisp games, and the gap below 0.002 in Table 2 supports the accuracy of such shortcuts, although only one benchmark was tested. The ranking indices of Yager [19], Chen [20] and Carlsson and Fullér [21] can order fuzzy numbers differently in small examples, whereas Table 5 shows that their effect on game values is small, with a maximum disagreement of 0.317 in 2,000 games. For cooperative games, Mallozzi et al. [14] and Alparslan Gök et al. [26] stress that convexity of the interval game is what guarantees a well-behaved core. Our crisp games are convex because the synergies are non-negative, and the core was always present; random fuzzification destroyed convexity in some games, which is consistent with the emptiness rates in Table 4. Finally, the linear law between spread and width (Table 3) and the strategy-change rates have, to our knowledge, not been reported numerically in the studies surveyed, so they are a new contribution of this work. Fuzzy goal models [29] and multiobjective extensions [8], [28] are outside the scope of this payoff-only experiment, so a direct numerical comparison with them is not possible. Overall, the findings are consistent with the analytical literature, add numerical frequencies to it, and suggest that the lower and upper games deserve routine attention. These links show that the experiment confirms known theory not contradicting it; this builds confidence in the model.

6. CONCLUSION

This paper examined three aspects of fuzzy game theory through a controlled computational study of 16,200 simulated games. The fuzzy value of a matrix game with triangular payoffs is an interval whose width grows linearly with the spread factor ($W \approx 5.58s$, $R^2 = 0.9998$) and does not depend on game size, although its variability shrinks by 66 % between 2×2 and 8×8 games. Triangular approximation of the value was accurate to within 0.002 in the benchmark, and the centroid, graded-mean and possibilistic-mean rules agreed with the crisp value to within 0.006 on average, with correlations above 0.99. Strategies and cooperative solutions, however, were less stable. The share of games whose optimal strategy support changed rose from 8.3 % to 33.4 % as s increased from 0.1 to 0.5, and fuzzification emptied the core in 15–23 % of lower-bound and 10–15 % of upper-bound cooperative games, while the Shapley allocation stayed within 1 % of the grand-coalition worth. These results lead to a practical recommendation: values may be defuzzified with any standard rule, but strategies and stable allocations

should be checked against the lower and upper games before they are adopted. Future work should test other fuzzy number shapes, non-zero-sum and multi-person games, correlated uncertainty, and real data from applications such as negotiation or resource sharing. The study is limited to simulated data, but the linear law and the strategy-instability rates provide benchmarks for applied studies. They can be reused by researchers who wish to compare new fuzzy solution concepts.

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